Combined Heat and Power Economic Dispatch Solution by Equilibrium Optimizer

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Abstract Due to its great economic benefit, the demand for utilizing combined heat and power (CHP) systems has increased. Many facilities and buildings need to be supplied with both energy and heat. Usually, CHP systems are used with other systems that only produce power or heat. The economic dispatch (ED) problem for the CHP systems, which seek to minimize the cost of fuel, is highly challenging. One of the most significant algorithms for solving the ED problem in CHP systems is the Equilibrium Optimizer (EO). This work examined the effectiveness of the EO in solving the ED in CHP systems. In this paper, the EO was applied to many different studied systems, taking into account the effect of losses and the effect of valve point in these different systems. The effectiveness of the EO would be compared to other algorithms applied in some of the previous works. The proposed algorithm (EO) has high-quality solutions and superior performance compared to some solutions presented recently.

Keywords: Combined Heat and Power; Economic Dispatch; Equilibrium Optimizer; Non-convex Optimization.

1 Introduction

Cogeneration systems, such as combined heat and power (CHP) systems, are the best way to produce two types of usable energy from a single fuel source[1]. In CHP systems, heat (steam) is the primary energy form, while electricity is the secondary energy source. CHP systems are usually used where there is a need for heat besides electrical power. To ensure meeting the required energy efficiencies and environmental requirements, CHP systems are often placed close to buildings where heat is needed.

The CHP Economic Dispatch (CHPED) issue aims to meet all the system heat and power demands at the minimum fuel cost while satisfying all the constraints. There are many algorithms that were used to get the optimal solution to the CHPED problem[2].

In this study, to solve the CHPED issue, equilibrium optimization (EO) was used. Different scales of test systems were used to evaluate the effectiveness of the suggested method. By comparing the results of the EO with some other previous techniques, the EO proved to be a reliable optimization method. Findings indicated that the EO can find the best solution to the CHPED problem.

2 Problem Formulation

The CHPED issue faced many challenges, such as the system having dual demands (heat and power) and the dependency of the capacity of the generated power on the capacity of the generated heat in the cogeneration units.

In the CHPED problem, there are three types of units: power-only, heat-only, and combined heat and power units. The objective function of CHPED is to minimize the operation cost of the system while satisfying the system constraints.

Many algorithms were used to solve the CHPED problem, such as: genetic algorithm (GA)[3], harmony search (HS)[4], Differential Evolution (DE)[5], improved ant colony search (IACS) [6], evolutionary programming (EP)[7], improved genetic algorithm with multiplier

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update (IGA-MU)[8], firefly algorithm (FA)[9], a self-adaptive real-coded genetic algorithm (SARGA)[10], exchange market algorithm (EMA)[11], time varying acceleration coefficients particle swarm optimization (TVAC–PSO)[12], Direct solution [13], integrated genetic - tabu search algorithm (G–ATS)[14], real coded genetic algorithm (RCGA)[15], Artificial Immune System (AIS) [16], Bee Colony Optimization (BCO)[17], Lagrangian relaxation[LR][18], benders decomposition (BD)[19], crisscross optimization algorithm (COA)[20], stochastic fractal search (SFS)[21], grasshopper optimization algorithm (GOA)[22], adaptive cuckoo search with differential evolution mutation (ACS-DEM)[23], group search optimization (GSO)[24], wild goats algorithm (WGA)[25], particle swarm optimization (PSO)[26], invasive weed optimization (IWO)[27], marine predators algorithm (MPOA)[28], artificial bee colony (ABC)[29], hybrid heap-based and jellyfish search algorithm (HBJSA)[30], real coded genetic algorithm with improved Mühlenbein mutation (RCGA-IMM)[15], weighted vertices-based optimizer (WVO)[31], etc.

3 Mathematical Modeling of the CHPED Problem

3.1 Objective Function

The formulation of the CHPED problem is given in [2]. The objective function is defined as:

\[
OF = \sum_{i=1}^{N_p} C_i(P_i^p) + \sum_{j=1}^{N_c} C_j(P_j^c, H_j^c) + \sum_{k=1}^{N_h} C_k(H_k^h) \tag{1}
\]

Where:

\[
C_i(P_i^p) = a_i((P_i^p)^2 + b_i P_i^p + c_i) \tag{2}
\]

\[
C_j(P_j^c) = a_j((P_j^c)^2 + b_j P_j^c + c_j + d_j \sin(e_j(P_j^{p_{min}} - P_j^c))) \tag{3}
\]

Where;

\(a_i, b_i, c_i\): Cost Function Coefficients of power only unit,
\(d_i, e_i\): cost functions of valve-point impacts,
\(P_i^{p_{min}}\): Lower Power out of power only unit,
\(P_i^c\): Power output of power only unit.

\[
C_i(P_i^p, H_i^h) = a_j((P_i^p)^2 + b_j P_i^p + c_j + d_j(H_i^h)^2 + e_j H_i^h + f_j P_i^p H_i^h \ (\$/h) \tag{4}
\]

Where;

\(a_j, b_j, c_j, d_j, e_j, f_j\): Cost Function Coefficients of Cogeneration Unit.

\[
C_k(H_k^h) = a_k(H_k^h)^2 + b_k H_k^h + c_k \ (\$/h) \tag{5}
\]

Where;

\(a_k, b_k, c_k\): Cost Function Coefficients of heat only unit.

3.2 Constraints

3.2.1 Power balanced Constraints:

The power generated from pure power units and combined units must meet the demand and lost capacity of lines[32].

\[
\sum_{i=1}^{N_p} P_i^p + \sum_{j=1}^{N_c} P_j^c = P_d \tag{6}
\]

Where;

\(P_d\): Power demand
\(P_i^p\): power output of only power unit.
\(P_j^c\): power output of cogeneration units.

3.2.2 Heat balance Constraints:

The total heat produced by pure heat units and cogeneration units must meet heat demand and neglected loss heat.

\[
\sum_{j=1}^{N_c} H_j^c + \sum_{k=1}^{N_h} H_k^h = Hd \tag{7}
\]

Where;

\(Hd\): Heat Demand,
\(H_j^c\): Heat output of cogeneration units,
\(H_k^h\): Heat output of heat only units.

3.2.3 Generation power and heat limits:

The produced electric and heat powers should be in the acceptable range for each unit:

\[
P_i^{p_{min}} \leq P_i^p \leq P_i^{p_{max}} \quad i = 1, 2, ..., N_p \tag{8}
\]

Where;

\(P_i^{p_{min}}, P_i^{p_{max}}\): lower & upper output power of power only units.

\[
P_j^{c_{min}}(H_j^c) \leq P_j^c \leq P_j^{c_{max}}(H_j^c) \quad j = 1, 2, ..., N_c \tag{9}
\]
Where;

\[ p_j^{\text{min}}, p_j^{\text{max}}: \text{lower & upper output power of cogeneration units.} \]

\[ H_j^{\text{min}}(P_j^c) \leq H_j^c \leq H_j^{\text{max}}(H_j^c) \]

\[ j = 1, 2, \ldots, N_c \]  

Where;

\[ H_j^{\text{min}}, H_j^{\text{max}}: \text{lower & upper heat output of cogeneration units.} \]

\[ H_k^h \leq H_k^h \leq H_k^{\text{max}} \]

\[ k = 1, 2, \ldots, N_h \]  

Where;

\[ H_i^h, H_k^h: \text{lower & upper heat output of heat only units.} \]

4 CHPED Formulation Through The EO

The EO algorithm, inspired by the control volume mass balance, is designed to estimate both dynamic and equilibrium states. EO falls into the third class of optimization algorithms, as it is derived from physical laws found in nature [33]. Inside the EO, each particle (solution) with its concentration (position) functions as a search agent. Search agents update their concentrations randomly according to the best-so-far solutions, known as equilibrium candidates, to eventually reach the state of equilibrium (optimal result). To enhance the EO algorithm's ability in exploration, exploitation, and local minima avoidance, a well-defined "generation rate" term has been proven effective. The mass-balanced equation is represented as:

\[ V \frac{dx}{dt} = QX_{eq} - QX + G \]  

Where, \( V \) is the control volume, \( X \) is the concentration, and \( Q \) the flow rate, \( V \frac{dx}{dt} \) is the rate of change of mass in the control volume, \( X_{eq} = X_{pool} \) represents the concentration at an equilibrium state in which there is no generation inside the control volume, \( G \) is the mass generation rate inside the control volume. The concentration \( X \) can be represented as follows:

\[ X = X_{eq} + (X_0 - X_{eq}) \exp[-\lambda(t - t_0)] + \frac{G}{AV} \left( 1 - (\exp[-\lambda(t - t_0)]) \right) \]  

Where;

\[ \lambda = \left( \frac{Q}{V} \right), X_0 \] denotes the initial concentration, while \( t_0 \) refers to the initial start time.

The following steps describe the procedure of the EO:

Step 1: Initialization

The concentrations are initialized randomly as follows:

\[ X_i^{\text{initial}} = X_{min} + \text{rand}_i(X_{max} - X_{min}) \]

\[ i = 1, 2, \ldots, n \]  

Where;

\( X_i^{\text{initial}} \) refers to the initial concentration vector of the \( i \)-th particle, \( X_{max} \) refers to the maximum limit of the control variables while \( X_{min} \) is the minimum limit. \( \text{rand}_i \) is a random variable within \([0,1]\). Then, evaluate the objective function for each concentration.

Step 2: Assigning the Equilibrium candidate's

The populations are sorting, and the four best solutions are captured and their average value to form the pool vector \( X_{pool} \) as follows:

\[ X_{avg} = \frac{X_1 + X_2 + X_3 + X_4}{4} \]  

\[ X_{pool} = \{X_1, X_2, X_3, X_4, X_{avg}\} \]  

Step 3: The concentration Update

Two randomly vectors \( (r, \lambda) \) are generated randomly and utilized to adjust an exponential factor \( F \) for updating the concentrations as follows:

\[ F = a_1 \text{sign}(r - 0.5) [e^{-\lambda t} - 1] \]  

\[ t = (1 - \frac{T}{T_{Max}}) (\frac{a_2 T}{T_{Max}}) \]  

Where;

\( F \) is the exponential term, \( a_1 \) and \( a_2 \) refer to constant...
terms, which are set to be 2 and 1, respectively, to adjust the exponential factor. $T_{\text{Max}}$ is the maximum iteration number, and $T$ denotes the $T$-th iteration. It should be indicated out here that $a_1$ is utilized to control the exploration phase of the EO and $a_2$ is utilized to control the exploitation process of the EO. $\text{Sign} (r - 0.5)$ controls the direction of the exploration.

**Step 4: Concentration updating based on the generation rate**

The generation rate is an efficient method to enhance the exploitation phase of the optimization algorithm as follows:

$$G = G_0 e^{-k(r-t_0)}$$ (19)

Where;

- $G_0$ is the initial value, $k$ indicates a decay constant,
- $r_1$ and $r_2$ are random numbers within the range $[0,1]$,
- $GCP$ is defined as the Generation rate Control Parameter,
- $GP$ denotes the generation probability to control the participation probability of concentration, which is updated by the generation rate. If $GP = 1$ generation rate

$$G_0 = GCP \times (X_{\text{pool}} - \lambda X)$$ (20)

$$GCP = \begin{cases} 0.5 \times r_1 & r_2 \geq GP \\ 0 & r_2 < GP \end{cases}$$ (21)

will be no participate in the optimization process. If $GP = 0$, the generation rate will participate in the process. $GP = 0.5$ provides an excellent balancing between exploitation and exploration phases. Referring to the previous steps, the updated equation of the EO is formulated as follows:

$$X = X_{\text{pool}} + (X - X_{\text{pool}}) \times F + \frac{G}{AV} (1 - F)$$ (22)

**Step 5: Adding memory saving**

In this step, the obtained concentration is compared with the previous concentration, and it will be accepted if this value is enhanced. Fig. 1 shows the flow chart of optimal energy management.

5 Results and discussion

This should include the findings of the study including, if appropriate, results of statistical analysis which must be included either in the text or as tables and figures. For research articles this section should discuss the implications of the findings in context of existing research and highlight limitations of the study. For methodology manuscripts this section should include a discussion of any practical or operational issues involved in performing the study and any issues not covered in other sections. The test problems taken into consideration are taken from Refs. [6, 34, 35]. For the cogeneration units, implausible solutions are rendered feasible while randomly generating candidate solutions by fixing them to the closest straight line in the contour. In the case of power-only & heat-only units, infeasible candidates are moved to the nearest upper or lower limits. The equality constraints are taken care of by the use of penalty functions augmenting the objective function. Simulations were conducted in MATLAB R2015a.

5.1 Tested System 1

A tested system of four units is taken to illustrate the performance of the proposed method. For the conventional power unit 1, For the cogeneration units 2 and 3, For the heat-only unit 4:

$$C_1 = 50P_1, \quad 0 \leq P_1 \leq 150 \text{ MW}$$ (23)

$$C_2 = 0.0345P_2^2 + 14.5P_2 + 2650 + 0.03H_2^2 + 4.2H_2 + 0.031P_2H_2$$ (24)

$$C_3 = 0.0345P_3^2 + 36P_3 + 1250 + 0.027H_3^2 + 0.6H_3 + 0.011P_3H_3$$ (25)

The power and heat demand for the system is 200MW and 115 MWth respectively. The heat-power feasible regions for the cogeneration units are illustrated in Figs. 2 & 3.

Fig. 4 shows the characteristics of System 1 which occurs after 500 iterations Table 1 shows the comparison between the results of the EO along with the other published results. In this system, by comparing the result obtained with the latest results, the EO saved an amount of money amounting to 0.03 $ / h; thus, the total annual saving is 262.8 $ / year.
Fig. 1 Flow chart of optimal energy management.

Fig. 2 Feasibility region for cogeneration unit 2 in System1.

Fig. 3 Feasibility region for cogeneration unit 3 in System1.


### Table 1
Comparison between the results of the EO along with the other published results.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>P&lt;sub&gt;1&lt;/sub&gt;</th>
<th>P&lt;sub&gt;2&lt;/sub&gt;</th>
<th>P&lt;sub&gt;3&lt;/sub&gt;</th>
<th>H&lt;sub&gt;2&lt;/sub&gt;</th>
<th>H&lt;sub&gt;3&lt;/sub&gt;</th>
<th>H&lt;sub&gt;4&lt;/sub&gt;</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACSA [36]</td>
<td>0.08</td>
<td>150.93</td>
<td>49</td>
<td>48.84</td>
<td>65.79</td>
<td>0.37</td>
<td>9452.2</td>
</tr>
<tr>
<td>GA [3]</td>
<td>0</td>
<td>159.23</td>
<td>40.77</td>
<td>39.94</td>
<td>75.06</td>
<td>0</td>
<td>9267.2</td>
</tr>
<tr>
<td>RGA [37]</td>
<td>0</td>
<td>158.18</td>
<td>41.82</td>
<td>37</td>
<td>78</td>
<td>0</td>
<td>9263.28</td>
</tr>
<tr>
<td>EP [7]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.1</td>
</tr>
<tr>
<td>FP [38]</td>
<td>0.0014</td>
<td>159.9986</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.1</td>
</tr>
<tr>
<td>HS [4]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
<tr>
<td>MU-IGA [8]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>39.99</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
<tr>
<td>SARGA [39]</td>
<td>0</td>
<td>159.99</td>
<td>40.01</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
<tr>
<td>EMA [11]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
<tr>
<td>TVAC–PSO [12]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
<tr>
<td>Direct method</td>
<td>[13]</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
<tr>
<td>The proposed algorithm (EO)</td>
<td>0</td>
<td>160</td>
<td>40</td>
<td>39.99</td>
<td>75</td>
<td>0</td>
<td>9257.07</td>
</tr>
</tbody>
</table>

### Fig. 4
Characteristics of the Tested System 1 (after 500 iterations).

#### 5.2 Tested System 2

The Tested System involved one conventional power unit, three cogeneration units, and a heat-only unit. For the conventional power unit:

\[
C_1 = 0.00115P_1^2 + 0.00172P_1^2 + 7.6997P_1 + 254.886 \\
35 \leq P_1 \leq 135 \text{ MW} \tag{26}
\]

For the cogeneration units 2, 3, and 4,

\[
C_2 = 0.0345P_2^2 + 36P_2 + 1250 + 0.027H_2^2 + 0.6H_2 + 0.011P_2H_2 \tag{27}
\]

\[
C_3 = 0.0345P_3^2 + 14.5P_2 + 2650 + 0.03H_3^2 + 4.2H_3 + 0.031P_2H_2 \tag{28}
\]

\[
C_4 = 0.072P_4^2 + 20P_4 + 1565 + 0.02H_4^2 + 2.3H_4 + 0.04P_4H_4 \tag{29}
\]

For the heat-only generated from unit 5,

\[
C_5 = 0.038H_5^2 + 2.0109H_5 + 950 \\
0 \leq H_5 \leq 60 \text{ MW} \tag{30}
\]

The heat-power feasible regions for the cogeneration units are illustrated in Figs. 5, 6 and 7.

### Fig. 5
Feasibility region for cogeneration unit 2 in System 2.
Combined Heat and Power Economic Dispatch Solution by Equilibrium Optimizer

Table 2 shows the comparison between the results of the EO along with the other published results. The power and heat demand for the system are 300MW and 150 MWth respectively Profile 1. By comparing the result obtained with the latest results, the EO saved an amount of money amounting to 6.119 $/h; thus, the total annual saving is 53602.44 $/year.

Table 2 Comparison between the EO results & the other published results for the system of 300MW and 150 MWth.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$P_1$ (MW)</th>
<th>$P_2$ (MW)</th>
<th>$P_3$ (MW)</th>
<th>$P_4$ (MW)</th>
<th>$H_2$ (MWth)</th>
<th>$H_3$ (MWth)</th>
<th>$H_4$ (MWth)</th>
<th>$H_5$ (MWth)</th>
<th>Min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA [3]</td>
<td>135.00</td>
<td>70.81</td>
<td>10.84</td>
<td>83.28</td>
<td>80.54</td>
<td>39.81</td>
<td>0.00</td>
<td>29.64</td>
<td>13,779.50</td>
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<tr>
<td>HS [4]</td>
<td>134.74</td>
<td>48.20</td>
<td>81.09</td>
<td>16.23</td>
<td>23.92</td>
<td>100.85</td>
<td>6.29</td>
<td>38.70</td>
<td>13,723.20</td>
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<td>CPSO [40]</td>
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<td>19.2728</td>
<td>105</td>
<td>64.4003</td>
<td>26.4119</td>
<td>0</td>
<td>39.3476</td>
<td>13,692.52</td>
</tr>
<tr>
<td>IWO [27]</td>
<td>134.73</td>
<td>40.00</td>
<td>75.00</td>
<td>20.86</td>
<td>37.60</td>
<td>104.41</td>
<td>0</td>
<td>37.40</td>
<td>13,683.65</td>
</tr>
<tr>
<td>FA [9]</td>
<td>134.74</td>
<td>40.05</td>
<td>20.15</td>
<td>105</td>
<td>70.85</td>
<td>26.4119</td>
<td>0</td>
<td>45.53</td>
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<tr>
<td>GSA [41]</td>
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<td>18.1736</td>
<td>105</td>
<td>74.089</td>
<td>37.3336</td>
<td>0</td>
<td>37.40</td>
<td>13,683.22</td>
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</table>

Proposed algorithm (EO) 135.00 40.00 20.54 104.45 77.25 27.23 0.00 56.76 13,671.14

Table 3 Comparison between the EO results & the other published results for the system of 250MW and 175 MWth.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$P_1$ (MW)</th>
<th>$P_2$ (MW)</th>
<th>$P_3$ (MW)</th>
<th>$P_4$ (MW)</th>
<th>$H_2$ (MWth)</th>
<th>$H_3$ (MWth)</th>
<th>$H_4$ (MWth)</th>
<th>$H_5$ (MWth)</th>
<th>Min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA [3]</td>
<td>119.22</td>
<td>45.12</td>
<td>15.82</td>
<td>69.89</td>
<td>78.94</td>
<td>22.63</td>
<td>18.4</td>
<td>54.99</td>
<td>12,327.37</td>
</tr>
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<td>HS [4]</td>
<td>134.67</td>
<td>52.99</td>
<td>10.11</td>
<td>52.23</td>
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<td>39.73</td>
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<td>CPSO [40]</td>
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<td>40.3446</td>
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<td>70.9318</td>
<td>39.9918</td>
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<td>IWO [27]</td>
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<td>38.98</td>
<td>8.81</td>
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<td>10</td>
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<td>40</td>
<td>16.97</td>
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<td>GSA [41]</td>
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<td>40</td>
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Proposed algorithm (EO) 135.00 40.05 10.06 64.87 78.48 39.74 0.00 56.76 12,111.47

Fig. 6 Feasibility region for cogeneration unit 3 in System 2.

Fig. 7 Feasibility region for cogeneration unit 4 in System 2.

Fig. 8 shows the characteristics of System 2 which occurs after 500 iterations profile 1.
Table 3 shows the comparison between the EO results &
the other published results, the power & heat demand for
the system are 250MW & 175 MWth respectively Profile
2. by comparing the result obtained with the latest
results, the EO saved an amount of money amounting
to 5.9 $ / h; thus, the total annual saving is 53602.44
$/year.
Fig. 9 shows the characteristics of System 2 which occurs
after 500 iterations profile 2.

5.3 Tested System 3

Data of this system are adopted from [5] Four
conventional thermal power units, two cogeneration units,
and a heat-only unit make up the tested system. In relation
to the standard thermal power units 1, 2, 3, and 4,

\[
C_1 = 0.008P_1^2 + 2P_1 + 25
+ [100 \sin(0.042(10
- P_1))]\]  

\[
C_2 = 0.003P_2^2 + 1.8 P_2 + 10
+ [140 \sin(0.04(20
- P_2))]\]  

\[
C_3 = 0.0012P_3^2 + 2.1P_3 + 100
+ [160 \sin(0.038(30
- P_3))]\]  

\[
C_4 = 0.001P_4^2 + 2P_4 + 120
+ [180 \sin(0.037 (40
- P_4))]\]  

For the cogeneration units 5 and 6,

\[
C_5 = 0.0345P_5^2 + 14.5P_5 + 2650
+ 0.03 H_5^2 + 4.2H_5
+ 0.031P_5H_5\]  

\[
C_6 = 0.0345P_6^2 + 36 P_6 + 1250
+ 0.027 H_6^2 + 0.6 H_6
+ 0.011P_6H_6\]  

For the heat-only unit 7,

\[
C_7 = 0.038 H_7^2 + 2.0109 H_5 + 950,
0 \leq H_7 \leq 60 \text{ MWth}\]  

\[
C_1 = 0.0028 P_1^2 + 8.1 P_1 + 550
+ [300 \sin(0.035(0
- P_1))]\]  

\[
0 < P_1 < 680\]  

\[
C_2 = 0.00056 P_2^2 + 8.1 P_2 + 309
+ [200 \sin(0.042 (0
- P_2))]\]  

\[
0 < P_2 < 360\]  

\[
C_3 = 0.00056 P_3^2 + 8.1 P_3 + 309
+ [200 \sin(0.042 (0
- P_3))]\]  

\[
0 < P_3 < 360\]  

\[
C_4 = 0.00324 P_4^2 + 7.74 P_4 + 240
+ [200 \sin(0.063 (60
- P_4))]\]  

\[
60 < P_4 < 180\]  

\[
C_5 = 0.00324 P_5^2 + 7.74 P_5 + 240
+ [200 \sin(0.063 (60
- P_5))]\]  

\[
0 < P_5 < 180\]  

The heat-power feasible regions for the cogeneration nits
are illustrated in Figs. 10 & 11.
In this section, the tested system considered both the valve-point effects and transmission losses to examine the effectiveness of the EO. Table 4 shows the comparison between the results of the EO along with some other published results, the power and heat demand were 600MW and 150 MWth respectively. By comparing the result obtained with the latest results, the EO saved an amount of money amounting to 50 $/h; thus, the total annual saving is 438000 $/year.

<table>
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<tr>
<th>Algorithm</th>
<th>$P_1$ (MW)</th>
<th>$P_2$ (MW)</th>
<th>$P_3$ (MW)</th>
<th>$P_4$ (MW)</th>
<th>$P_5$ (MW)</th>
<th>$H_3$ (MWth)</th>
<th>$H_4$ (MWth)</th>
<th>$H_5$ (MWth)</th>
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<td>100.9363</td>
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<td>209.8158</td>
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<td>94.1838</td>
<td>40</td>
<td>27.1786</td>
<td>75</td>
<td>47.8214</td>
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<tr>
<td>Proposed algorithm (EO)</td>
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<td>98.5398</td>
<td>112.6734</td>
<td>209.8158</td>
<td>94.1838</td>
<td>40</td>
<td>27.1786</td>
<td>75</td>
<td>47.8214</td>
</tr>
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</table>

Fig. 12 shows the characteristics of System 3 which occurs after 500 iterations.

Fig. 13 Characteristics of the Tested System 3 (after 500 iterations).

5.4 Tested System 4

A large-scale test system consists of 24 units is considered. Among twenty-four units, units 1–13 are power-only units, 14–19 are cogeneration units and 20–24 are heat-only units. The fuel cost function includes valve point effects, The power and heat demand for the system is 2350MW and 1250 MWth respectively. In relation to the standard thermal power units 1-13, For the conventional thermal power units,

\[
C_6 = 0.00324 P_6^2 + 7.74 P_6 + 240 + [200 \sin(0.063 \times 60 - P_6)] \quad (\$/h) \quad (43)
\]

\[
0 < P_6 < 180
\]

\[
C_7 = 0.00324 P_7^2 + 7.74 P_7 + 240 + [200 \sin(0.063 \times 60 - P_7)] \quad (\$/h) \quad (44)
\]

\[
0 < P_7 < 180
\]
\[ C_8 = 0.00324 \, P_8^2 + 7.74 \, P_8 + 240 + [200 \, \sin(0.063 \, (60 - P_8)) \] \] ($/h$) \[ 0 < P_8 < 180 \]

\[ C_9 = 0.00324 \, P_9^2 + 7.74 \, P_9 + 240 + [200 \, \sin(0.063 \, (60 - P_9)) \] \] ($/h$) \[ 0 < P_9 < 180 \]

\[ C_{10} = 0.00284 \, P_{10}^2 + 8.6 \, P_{10} + 126 + [100 \, \sin(0.084 \, (40 - P_{10})) \] \] ($/h$) \[ 40 < P_{10} < 120 \]

\[ C_{11} = 0.00284 \, P_{11}^2 + 8.6 \, P_{11} + 126 + [100 \, \sin(0.084 \, (40 - P_{11})) \] \] ($/h$) \[ 40 < P_{11} < 120 \]

\[ C_{12} = 0.00284 \, P_{12}^2 + 8.6 \, P_{12} + 126 + [100 \, \sin(0.084 \, (55 - P_{12})) \] \] ($/h$) \[ 40 < P_{12} < 120 \]

\[ C_{13} = 0.00284 \, P_{13}^2 + 8.6 \, P_{13} + 126 + [100 \, \sin(0.084 \, (55 - P_{13})) \] \] ($/h$) \[ 40 < P_{13} < 120 \]

For the cogeneration units,

\[ C_{14} = 0.0345 \, P_{14}^2 + 14.5 \, P_{14} + 2650 + 0.03 \, H_{14}^2 + 4.2 \, H_{14} + 0.031 \, P_{14}H_{14} \] ($/h$) \[ 40 < P_{14} < 120 \]

\[ C_{15} = 0.0435 \, P_{15}^2 + 36 \, P_{15} + 1250 + 0.027 \, H_{15}^2 + 0.6 \, H_{15} + 0.011 \, P_{15}H_{15} \] ($/h$) \[ 40 < P_{15} < 120 \]

\[ C_{16} = 0.0345 \, P_{16}^2 + 14.5 \, P_{16} + 2650 + 0.03 \, H_{16}^2 + 4.2 \, H_{16} + 0.031 \, P_{16}H_{16} \] ($/h$) \[ 40 < P_{16} < 120 \]

\[ C_{17} = 0.0435 \, P_{17}^2 + 36 \, P_{17} + 1250 + 0.027 \, H_{17}^2 + 0.6 \, H_{17} + 0.011 \, P_{17}H_{17} \] ($/h$) \[ 40 < P_{17} < 120 \]

\[ C_{18} = 0.1035 \, P_{18}^2 + 34.5 \, P_{18} + 2650 + 0.025 \, H_{18}^2 + 2.203 \, H_{18} + 0.051 \, P_{18}H_{18} \] ($/h$) \[ 40 < P_{18} < 120 \]

\[ C_{19} = 0.072 \, P_{19}^2 + 20 \, P_{19} + 1565 + 0.02 \, H_{19}^2 + 2.34 \, H_{19} + 0.040 \, P_{19}H_{19} \] ($/h$)

For the heat-only units,

\[ C_{20} = 0.038 \, h_{20}^2 + 2.0109 \, h_{20} + 950 \] ($/h$) \[ 0 < P_{20} < 2695.20 \]

\[ C_{21} = 0.038 \, h_{21}^2 + 2.0109 \, h_{21} + 950 \] ($/h$) \[ 0 < P_{21} < 60 \]

\[ C_{22} = 0.038 \, h_{22}^2 + 2.0109 \, h_{22} + 950 \] ($/h$) \[ 0 < P_{22} < 60 \]

\[ C_{23} = 0.052 \, h_{23}^2 + 3.0651 \, h_{23} + 480 \] ($/h$) \[ 0 < P_{23} < 120 \]

\[ C_{24} = 0.052 \, h_{24}^2 + 3.0651 \, h_{24} + 480 \] ($/h$) \[ 0 < P_{24} < 120 \]

Comparing the EO with others; it can be noted that the EO is the lowest in cost, as the cost of generating energy and heat reaches 57920.26$. Data of this system are adopted from [2], published results. Table 5 shows the comparison between the results of the EO along with the other published results. The power and heat demand for the system is 2350MW and 1250 MW h respectively. The heat-power feasible regions for the cogeneration units are illustrated in Figs. 13,14,15 & Fig. 16 shows the characteristics of System 4 which occurs after 500 iterations. by comparing the result obtained with the latest results, the EO saved an amount of money amounting to 86.73 $ / h; thus, the total annual saving is 759754.8 $ /year.

![Fig. 13. Feasibility region for cogeneration unit 14,16 in System4.](image-url)
Table 5 Comparison between the EO results & the other published results, for the system are 235MW and 1250 MWh respectively.

<table>
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<th></th>
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<th></th>
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</tr>
</thead>
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<td>$P_1 (MW)$</td>
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<td>628.324</td>
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<td>299.4778</td>
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<td>60.846</td>
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Fig. 14. Feasibility region for cogeneration unit 15,17, in System 4.

Fig. 15. Feasibility region for cogeneration unit 19, in System 4.
6 Conclusions

CHP systems generate both electricity and thermal energy with high efficiency. The economic dispatch problem aims to meet power and heat demands at the lowest possible cost. This work studied different tested systems. The EO was used to solve the economic dispatch problem in these systems, and its effectiveness was compared with that of other algorithms. The EO provided the best solution among the algorithms compared.

7 References


[26] K. Kazda and X. Li, "A critical review of the modeling and optimization of combined heat and power dispatch,"


